



# Influenced of Yarn Structure for Predicting Quality of Yarn Based on Euclidean Coordinate (Theoretically and Experimentally)

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**Abstract:** In industry, commonly yarn can be made and produced by OE Spinning machine. In general, yarn structure is used to determine the quality of yarn, such as yarn count number and yarn strength. Based on theoretical consideration, in this research, a yarn can be analysed by the movement of fibre. According to this research, yarn structure was modelled on torus coordinate influenced by internal force known as stress.

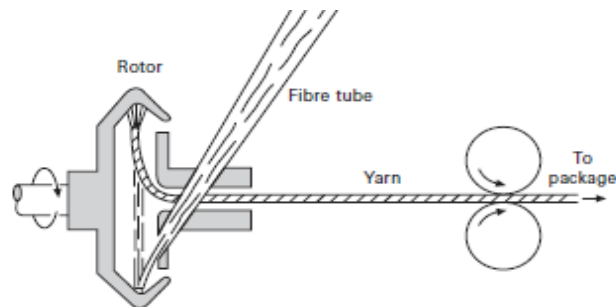
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## 1. Introduction

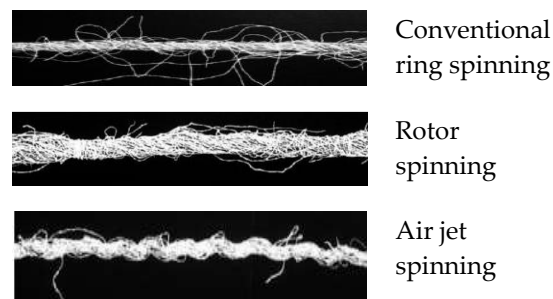
The application of classical mechanics to study material movement and its influence on properties of material has been studied by many researchers [1-21]. Lord [9] said that open end spinning, also referred as OE spinning, is a method for making yarn by OE rotor spinning which individual fibre is collected and twisted into the yarn structure (basic features of OE rotor spinning can be shown on Figure.1 below). In the process of making yarn, there are several variables to determine yarn count such as: twist  $T$  defined as the turn of fibres per length,  $F_{take-off} = F_o$  defined as the tension in the take-off nozzle inside rotor,  $\alpha$  as the angle of twist. Yarn structure can be analysed using several methods. The theories of yarn structure can be found and studied in several papers and some books as [1], [2], [3], [4], [5], [17], [18] and [19]. According to

Rohlena <sup>[17]</sup>, Putra et.al. <sup>[20,21]</sup>, Lawrence <sup>[4,5]</sup> and Backer et al <sup>[1]</sup>, twist is defined as the ratio of rotor angular speed to delivery yarn speed. In general, by knowing the yarn structure, the magnitude of twist used for predicting the yarn count number and the strength of yarn can be explained.



**Figure 1.** Basic Feature of OE Rotor Spinning Machine

Several researchers <sup>[1],[2],[3]</sup> and <sup>[5]</sup> formulated the yarn structure based on cylindrical coordinate or helical model. The yarn structure based on the helical model is called the migration theory. Fibre migration is the change in the distance of a fibre (along its length) from the axis of a yarn, which occurs during production of the open end spinning yarn (OE yarn). According to Lawrence <sup>[5]</sup>, yarn structure can be differenced by the spinning machine which is related by the fibre movement inside of yarn during the process of making yarn. The difference of yarn structure based on the spinning machine characteristic can be shown on the Figure 2.



**Figure 2.** Structure of Yarn Based on Machine Characteristic ( Lawrence, 2010)

According to all researchers <sup>[1],[2],[3],[4],[5],[6],[17],[18]</sup> yarn structure will influence the yarn count number and yarn strength. Based on the experimental result, Lotka <sup>[8]</sup> and Backer et al., <sup>[1]</sup> said that the strength of yarn is influenced by the rate of twist and the relation is shown as the lower of twist, the higher of the strength of yarn per tex and vice versa. Rohlena <sup>[17]</sup> said that breakage rate is influenced by the twist. The lower the twist, the lower the breakage rate. According to Musa <sup>[14]</sup>, Penava <sup>[15]</sup> and Prendzova <sup>[16]</sup>, the strength of yarn and yarn count number are affected by the yarn diameter. Musa <sup>[14]</sup> said that The wider yarn diameter , the stronger the yarn. According to Backer et al., <sup>[1]</sup> fibre migration in yarn results from

the interaction of two mechanisms: one depending on the stress of yarn and the other one depending on the magnitude of twist. In this research, the analysis given in this method assume the idealized yarn and ignore the interaction of each yarn during the movement of fibre inside the yarn.

## 2. Fibre Movement on Torus Coordinate

The mathematics of curved space is known as Riemannian geometry. In order to make a new model of theoretical analysis for fibre movement on torus coordinate, the transformation of coordinate which is suitable with the coordinate must be determined and decided. According to Margenau <sup>[11]</sup>, Moore <sup>[13]</sup>, Mal <sup>[10]</sup>, Martin[12] and Levrino[7], the transformation or the mapping of a certain coordinate to other coordinate is assumed as one-to-one, invertible, continuous and the Jacobian of mapping should be nonzero for all element of reference coordinate (positive definite). In the case of a first-rank tensor, the tangent vector  $C_i$  and the transformed vector  $\tilde{C}_m$  related to the tangent vector  $C_i$  by the equation (1) shall be considered.

$$\tilde{C}_\mu = \frac{dx^m}{d\tilde{x}^\mu} C_m \quad (1)$$

Consider a transformation from Cartesian coordinate to Torus coordinate as below

$$S = (x, y, z) = ((b + r \cos v) \cos u, (b + r \cos v) \sin u, r \sin v) \quad (2)$$

The tangent vector of the mapping of Cartesian coordinate to Torus coordinate can be shown in equation (3) below:

$$\begin{pmatrix} \tilde{C}_1 \\ \tilde{C}_2 \\ \tilde{C}_3 \end{pmatrix} = \begin{pmatrix} -\sin u(b + r \cos v) & \cos u(b + r \cos v) & 0 \\ -r \sin v \cos u & -r \sin v \sin u & r \cos v \\ \cos v \cos u & \cos v \sin u & \sin v \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \quad (3)$$

The unit vector of the tangent vector ( based vector) can be shown as equation (4):

$$\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{r} \end{pmatrix} = \begin{pmatrix} -\sin u & \cos u & 0 \\ -\sin v \cos u & -\sin v \sin u & \cos v \\ \cos v \cos u & \cos v \sin u & \sin v \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \quad (4)$$

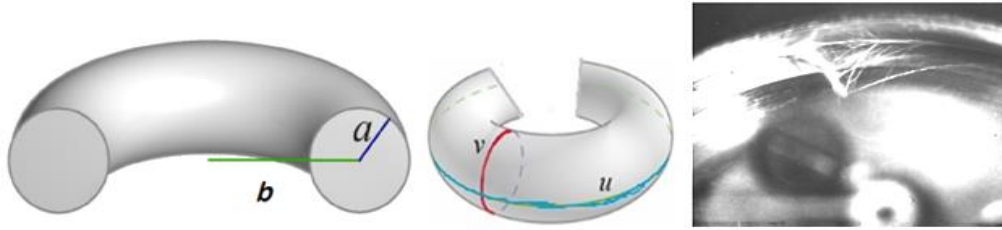
The square of the line element of Torus can be described as below:

$$dS^2 = (dx^2 + dy^2 + dz^2) = ((b + r \cos v)^2 du^2 + r^2 dv^2 + dr^2) \quad (5)$$

The metric tensor can be formulated as:

$$g_{mn} = \begin{pmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{pmatrix} = \begin{pmatrix} (b + r \cos v)^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (6)$$

In particular, fibre, with a density of mass  $\rho = m$  per each unit volume moves inside a yarn. The position of fibre can be determined by  $du, dv$  and  $dr$ . The rotor influenced the yarn rotation measured as  $n_r = \dot{v}$ . The yarn delivery speed is determined by the  $v_d = \dot{u}r$ . The fibre moves along the yarn during a time  $dt$ . A yarn is assumed to be formed as Torus coordinate which radius is  $r = a$  and the length of gap  $b$  (Figure.3)



**Figure 3.** The Movement of Fibre inside Yarn

The Christoffel symbols are given by:

$$\begin{aligned} \Gamma^1_{11} &= g^{11} \frac{1}{2} [d_1 g_{11} + d_1 g_{11} - d_1 g_{11}] = 0 \\ \Gamma^1_{12} = \Gamma^1_{21} &= \frac{g^{11}}{2} d_2 g_{11} = -\frac{r \sin v}{b + r \cos v} \\ \Gamma^1_{22} &= g^{11} \frac{1}{2} [d_2 g_{21} + d_2 g_{12} - d_1 g_{22}] = 0 \\ \Gamma^1_{13} = \Gamma^1_{31} &= g^{11} \frac{1}{2} d_3 g_{11} = \frac{\cos v}{b + r \cos v} \\ \Gamma^1_{33} = \Gamma^1_{32} &= \Gamma^1_{23} = 0 \end{aligned}$$

$$\begin{aligned} & \frac{d^2 x^1}{dt^2} + \Gamma_{\alpha\beta}^1 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \\ &= \frac{d^2 x^1}{dt^2} + \Gamma_{11}^1 \frac{dx^1}{dt} \frac{dx^1}{dt} + \Gamma_{12}^1 \frac{dx^1}{dt} \frac{dx^2}{dt} + \Gamma_{13}^1 \frac{dx^1}{dt} \frac{dx^3}{dt} + \Gamma_{21}^1 \frac{dx^2}{dt} \frac{dx^1}{dt} + \Gamma_{22}^1 \frac{dx^2}{dt} \frac{dx^2}{dt} \\ &+ \Gamma_{23}^1 \frac{dx^2}{dt} \frac{dx^3}{dt} + \Gamma_{31}^1 \frac{dx^3}{dt} \frac{dx^1}{dt} + \Gamma_{32}^1 \frac{dx^3}{dt} \frac{dx^2}{dt} + \Gamma_{33}^1 \frac{dx^3}{dt} \frac{dx^3}{dt} \\ &= \frac{d^2 u}{dt^2} - \frac{2r \sin v}{b + r \cos v} \frac{du}{dt} \frac{dv}{dt} + \frac{2 \cos v}{b + r \cos v} \frac{du}{dt} \frac{dr}{dt} = a^u \end{aligned} \quad (8)$$

$$\begin{aligned} & \frac{d^2 x^2}{dt^2} + \Gamma_{\alpha\beta}^2 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} \\ &= \frac{d^2 x^1}{dt^2} + \Gamma_{11}^2 \frac{dx^1}{dt} \frac{dx^1}{dt} + \Gamma_{12}^2 \frac{dx^1}{dt} \frac{dx^2}{dt} + \Gamma_{13}^2 \frac{dx^1}{dt} \frac{dx^3}{dt} + \Gamma_{21}^2 \frac{dx^2}{dt} \frac{dx^1}{dt} + \Gamma_{22}^2 \frac{dx^2}{dt} \frac{dx^2}{dt} \\ &+ \Gamma_{23}^2 \frac{dx^2}{dt} \frac{dx^3}{dt} + \Gamma_{31}^2 \frac{dx^3}{dt} \frac{dx^1}{dt} + \Gamma_{32}^2 \frac{dx^3}{dt} \frac{dx^2}{dt} + \Gamma_{33}^2 \frac{dx^3}{dt} \frac{dx^3}{dt} \\ &= \frac{d^2 v}{dt^2} + \frac{1}{r} \sin v (b + r \cos v) \left( \frac{du}{dt} \right)^2 + \frac{2}{r} \frac{dv}{dt} \frac{dr}{dt} = a^v \end{aligned} \quad (9)$$

$$\frac{d^2x^3}{dt^2} + \Gamma_{\alpha\beta}^3 \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt} = \frac{d^2r}{dt^2} + \cos v (b + r \cos v) \left(\frac{du}{dt}\right)^2 - r \left(\frac{dv}{dt}\right)^2 = a^r \quad (10)$$

$$\nabla \cdot \vec{\sigma} + \bar{F} = m\bar{a} \quad (11)$$

$$\begin{aligned} \frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b + r \cos v} \left( \frac{\partial \sigma^{ur}}{\partial u} \right) + \frac{1}{r} \left( \frac{\partial \sigma^{vr}}{\partial v} \right) + \frac{\sigma^{rr} \cos v - \sigma^{vr} \sin v}{b + r \cos v} + \frac{\sigma^{rr}}{r} - F_o \\ = m \left( \frac{d^2 r}{dt^2} + \cos v (b + r \cos v) \dot{u}^2 - r \dot{v}^2 \right) \end{aligned} \quad (12)$$

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$$\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b} \left( \frac{\partial \sigma^{ur}}{\partial u} \right) + \frac{1}{r} \left( \frac{\partial \sigma^{vr}}{\partial v} \right) - \frac{\sigma^{vr}}{b} + \frac{\sigma^{rr}}{r} - F_o = -mr\dot{v}^2 \quad (13)$$

$$\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b} \left( \frac{\partial \sigma^{ur}}{\partial u} \right) + \frac{1}{r} \left( \frac{\partial \sigma^{vr}}{\partial v} \right) - \frac{\sigma^{vr}}{b} + \frac{\sigma^{rr}}{r} - F_o = -ml \tan \left( \frac{\theta}{2} \right) \dot{v}^2 \quad (14)$$

In the case of  $\sigma^{ii} = \sigma^o$  is constant and the  $\sigma^{ij} = 0$  ( no friction effect is occurred ), then

$$F_o - \frac{\sigma^{rr}}{r} = ml \tan \left( \frac{\theta}{2} \right) \dot{v}^2 \quad (15)$$

Consider the magnitude of internal force is much less than the take-off force ,  $\frac{\sigma^{rr}}{r} \ll F_o$  hence

$$F_o - \frac{\sigma^{rr}}{r} \approx F_o = ml \tan \left( \frac{\theta}{2} \right) \dot{v}^2 \quad (16)$$

$$l = (R_r \tan \theta) = \frac{2F_o}{\sin \theta m \dot{v}^2} \quad (17)$$

$$\tan \theta \approx \sqrt{\frac{2F_o}{R_r m \dot{v}^2}} = 1,41 \frac{\sqrt{F_o}}{R_r \omega} \sqrt{N_m} \quad (18)$$

$$\tan \theta \approx 1,41 \frac{\sqrt{F_o}}{R_r \omega} \sqrt{N_m} = 1,41 \frac{\sqrt{F_o}}{R_r 2\pi n_r} \sqrt{N_m} = 1,41 \frac{\sqrt{F_o}}{n_r \pi d} \sqrt{N_m} \quad (19)$$

According to all researcher [1],[18], and [2], twist  $T$  is defined as below:

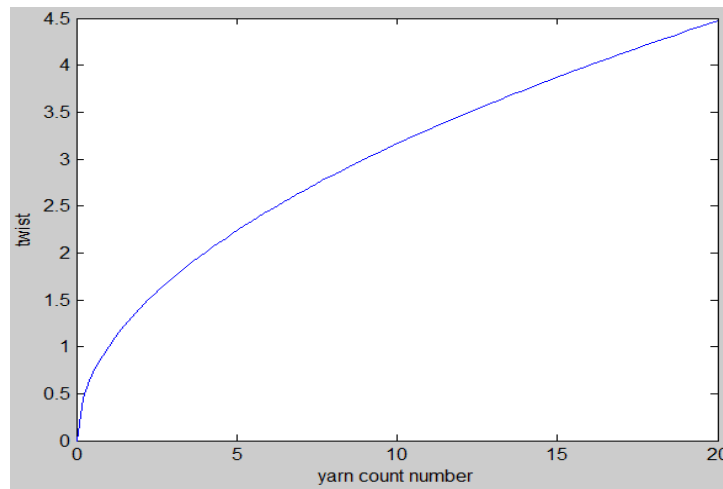
$$T = \frac{\tan \theta}{2\pi r_{yarn}} = \frac{\tan \theta}{\pi d_{yarn}} \quad (20)$$

Hence by substituting equation (19) to equation (20), it can be shown

$$T = \frac{1,41 \sqrt{F_o}}{n_r \pi^2 d_{yarn} d} \sqrt{N_m} \quad (21)$$

$$T = \left( \frac{2}{n_r \pi^{5/2} d} \sqrt{\frac{F_o}{\rho}} \right) \frac{1}{d_{yarn}^2} \quad (22)$$

From the equation (21), the relation of twist and yarn count can be graphed number as below:



**Figure 4.** Relation of Twist and Yarn Count Number  
( based on theoretical consideration)

According to the experiment result in Industry by following data below (**Table-1** and **Table-2**)

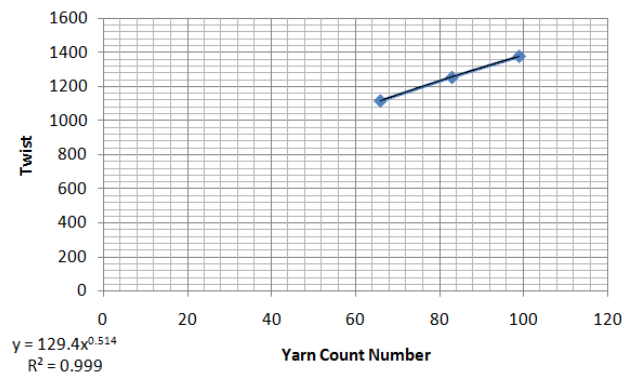
**Table.1** Experimental Result in Industry

rotor-speed	$a_e$	$a_m$	$N_e$	$N_m$	$V_d$	$T$	$H/m$
					(m/min)	(tpm)	
7200	4.47	135.454545	39.37	66.72881	64.27273	1119.75	3.46
7200	4.5	136.363636	49	83.05085	57.27273	1256.54	2.25
7200	4.5	136.363636	58.84	99.72881	52.18182	1379.31	2.13

**Table.2** Experimental Result in Industry

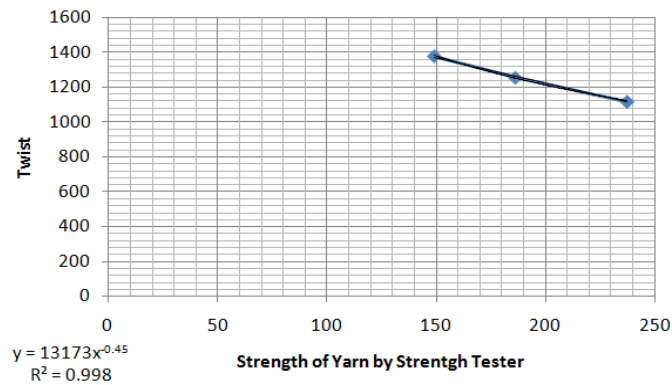
$Tt (tex)$	$v_d(yard/min)$	$Strenght (cN)$	$H/100yds$
14,98	70.7	237	315
12,04	63	186	205
10,10	57.4	149	194

By using Table 1 and Table 2 above, the relationship of Twist and Yarn Count in metric  $N_m$  can be made, as shown in Figure5 below:



**Figure 5.** Relation of Twist and Yarn Count Number (experimentally)

The relation of twist and yarn strength can be shown as Figure 6 below:



**Figure 6.** Relation of Twist and Strength of Yarn

### 3. Results and Discussion

Based on the theoretical consideration as well as experimental approach, it can be found that the higher twist, the higher is yarn count number. The higher yarn diameter the lower is the twist as well as yarn count number. In this research, it has been found and determined that the formula to determine the relationship of yarn count number and twist can be used in equation (21). According to Musa [4], Penava [5] and Prendzova [6], the strength of yarn and yarn count number are influenced by the yarn diameter. Musa [4] said that the wider yarn diameter, the stronger the yarn. Based on this experimental research, the lower the twist is, the higher the strength of yarn and vice versa. According to equation (22) it is said that the higher yarn diameter the lower the twist, hence it can be concluded that the yarn diameter will affect the strength as said by Musa [14]. In this new model, the relationship of twist and yarn count number which is similar pattern as the pattern by experiment has been shown in Figure 4. The



mathematics of curved space is known as Riemannian geometry can be used to determine the movement of fibre inside a yarn.

The new model of theoretical analysis for fibre movement on torus coordinate has been determined and formulated. According to Margenau [11], Moore[13], Mal[10], Martin[12] and Levrino[7], the transformation or the mapping of a certain coordinate to other coordinate is assumed as one-to-one, invertible, continuous and the Jacobian of mapping should be nonzero for all element of reference coordinate (positive definite) shown in equation (3) and equation (4). The new model shows a good result for predicting the yarn structure used to determine the quality of yarn, such as yarn count number and yarn strength.

#### 4. Conclusion

The structure of yarn on torus coordinate has been analysed to predict the relationship of yarn count in metric to yarn twist. In this research, it has been initiated that yarn twist is influenced by yarn count number in metric on Torus coordinate. It has been found that the formula to relate the relationship is found in equation (21)

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#### Nomenclature

Symbol	Dimension	Description
$T$	$[L]^{-1}$	Twist (tpm)
$n_r$	$[T]^{-1}$	Angular speed of rotor
$d_{yarn}$	$[L]$	Diameter of yarn
$d$	$[L]$	Diameter of rotor
$R_r$	$[L]$	Radius of rotor
$\rho$	$[M][L]^{-3}$	Volume density of yarn
$l$	$[L]$	Length of fibre in one turn
$F_o$	$[M][L][T]^{-2}$	External force occurred by rotor
$\sigma^{ij}$	$[M][T]^{-2}[L]^{-1}$	Yarn internal stress tensor
$T_t = \lambda$	$[M][L]^{-1}$	Yarn count number in tex (g/km)
$\theta$		Twist angle of yarn
$N_m$	$[L][M]^{-1}$	Yarn count number in metric (m/g)

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